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ONE-VELOCITY MODEL OF A MULTICOMPONENT HEAT-CONDUCTING MEDIUM

V. S. Surov

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A model of a one-velocity heat-conducting heterogeneous medium with the Fourier relaxation law of heat transfer has been constructed. It is shown that the model's equations are of hyperbolic type. The results of numerical experiments for a three-component mixture of ideal gases carried out with the use of the Courant–Isaacson–Rees scheme are presented.

Keywords: *one-velocity multicomponent heat conducting medium, Fourier heat transfer relaxation law, hyperbolic systems of nondivergent form, numerical simulation.*

Introduction. It is known that on immediate release of heat in a finite region a change in temperature in the entire space also occurs instantly (see [1]), which is incorrect from the physical point of view. In [2], this problem was discussed and a method to solve this paradox was proposed which was attributed to the use of explicit finite-difference schemes and application of minimum scales of quantities. A different approach to the solution of the problem on instantaneous propagation of heat was considered in [3], where instead of the ordinary Fourier law the generalized law of heat transfer allowing for the heat flux relaxation was used. In the case of a heat-conducting gas, the system of equations with the generalized Fourier heat-transfer law becomes hyperbolic, and therefore the laws of propagation of both gas-dynamical and thermal waves cannot be infinite.

In the present work, based on the one-velocity model of a heterogeneous medium [4] that allows for the interfractional-interaction forces, a hyperbolic model of a multicomponent heat-conducting medium with the only carrying fraction has been constructed.

One-Velocity Multicomponent Mixture. Let us consider an n-component mixture with the first m compressible fractions. The equations that describe the flow of a heterogeneous medium and which allow for the forces of interfractional interaction have the form (see [4])

$$\frac{\partial \rho}{\partial t} + \text{div}\,(\rho \mathbf{u}) = 0\,, \quad \rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\,\mathbf{u}\right] + \text{grad}\, p = \mathbf{F}\,;$$

$$\frac{\partial}{\partial t}\left[\rho\left(\varepsilon + \frac{1}{2}\,|\mathbf{u}|^2\right)\right] + \text{div}\left[\rho\left(\varepsilon + \frac{1}{2}\,|\mathbf{u}|^2 + \frac{p}{\rho}\right)\mathbf{u} + \mathbf{W}\right] = \mathbf{F}\cdot\mathbf{u}\,; \quad \frac{\partial \alpha_i \rho_i^0}{\partial t} + \text{div}\,(\alpha_i \rho_i^0 \mathbf{u}) = \sum_{k=1}^{n(k\neq i)} J_{ik}\,; \tag{1}$$

$$\rho_i\left(\frac{\partial \varepsilon_i}{\partial t} + (\mathbf{u}\cdot\nabla)\,\varepsilon_1\right) + \frac{\alpha_i p}{\rho_i}\left[\sum_{k=1}^{n(k\neq i)} J_{ik} - \left(\frac{\partial \rho_i}{\partial t} + (\mathbf{u}\cdot\nabla)\,\rho_i\right)\right] + \text{div}\,(\alpha_i \mathbf{W}_i)$$

$$= \sum_{k=1}^{n(k\neq i)} (R_{ik} + Q_{ik}) - \left(\varepsilon_i - \frac{1}{2}\,|\mathbf{u}|^2\right)\sum_{k=1}^{n(k\neq i)} J_{ik}\,, \quad i = 1, ..., m-1\,;$$

South-Urals State University, 76 Lenin Ave., Chelyabinsk, 454080, Russia; email: svs@csu.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 83, No. 1, pp. 132–141, January–February, 2010. Original article submitted October 9, 2008.

$$\frac{\partial \alpha_j}{\partial t} + \mathrm{div}\,(\alpha_j \mathbf{u}) = \frac{1}{\rho_j^0} \sum_{k=1}^{n(k\neq j)} J_{jk}\,, \quad j = m+1, ..., n\,.$$

System (1) is considered together with the generalized Fourier heat-transfer law allowing for thermal relaxation for the averaged heat flux:

$$\eta \left(\frac{\partial \mathbf{W}}{\partial t} + (\mathbf{u}\cdot\nabla)\,\mathbf{W} \right) + \chi\,\mathrm{grad}\,T + \mathbf{W} = 0\,. \tag{2}$$

At a zero value of the coefficient of thermal relaxation expression (2) coincides with the ordinary Fourier law.

The behavior of compressible fractions is described by the caloric equations of state $\varepsilon_i = \varepsilon_i(p, \rho_i^0)$; therefore the expression for the specific internal energy of the mixture can be written as

$$\varepsilon = \varepsilon\,(\rho, p, \alpha_1, \rho_1^0, ..., \alpha_{m-1}, \rho_{m-1}^0, \alpha_{m+1}, ..., \alpha_n)\,. \tag{3}$$

The average temperature is defined by the equation

$$T = \sum_{i=1}^{n} \alpha_i T_i\,, \tag{4}$$

where T_i is the local temperature of the ith fraction, which can be found from the thermal equation of state $T_i = T_i(p, \rho_i^0)$. Allowing for the equalities $\sum_{i=1}^{n} \alpha_i = 1$ and $\rho = \sum_{i=1}^{n} \rho_i$, we rewrite Eq. (4) as

$$T = T\,(\rho, p, \alpha_1, \rho_1^0, ..., \alpha_{m-1}, \rho_{m-1}^0, \alpha_{m+1}, ..., \alpha_n)\,. \tag{5}$$

We assume that in the mixture the only fraction filling the space in a cohesive way is the mth component. All the rest fractions are "disseminated" in the mth (carrying) one; therefore in the equations of heat influx of system (1) we must omit terms of the form of div $(\alpha_i \mathbf{W}_i)$ responsible for heat transfer in the ith $(i \neq m)$ fractions.

Upon transformation (see [4]), the system of governing equations (1)–(2) in the absence of mass forces, phase and chemical conversions, as well as of heat transfer by radiation for one-dimensional plane flows, is reduced to the form

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} = 0\,, \quad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0\,, \quad \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + \rho c^2\frac{\partial u}{\partial x} + H\frac{\partial W}{\partial x} = 0\,,$$

$$\frac{\partial W}{\partial t} + u\frac{\partial W}{\partial x} + k_\rho\frac{\partial \rho}{\partial x} + k_p\frac{\partial p}{\partial x} + \sum_{i=1}^{m-1}\left(k_{\alpha_i}\frac{\partial \alpha_i}{\partial x} + k_{\rho_i^0}\frac{\partial \rho_i^0}{\partial x}\right) + \sum_{j=m+1}^{n} k_{\alpha_j}\frac{\partial \alpha_j}{\partial x} + \frac{1}{\eta}W = 0\,; \tag{6}$$

$$\frac{\partial \rho_i^0}{\partial t} + u\frac{\partial \rho_i^0}{\partial x} + \rho_i^0 G_i\frac{\partial u}{\partial x} = 0\,, \quad \frac{\partial \alpha_i}{\partial t} + u\frac{\partial \alpha_i}{\partial x} + \alpha_i\,(1 - G_i)\frac{\partial u}{\partial x} = 0\,, \quad i = 1, ..., m-1\,;$$

$$\frac{\partial \alpha_j}{\partial t} + u\frac{\partial \alpha_j}{\partial x} + \alpha_j\frac{\partial u}{\partial x} = 0\,, \quad j = m+1, ..., n\,,$$

where

$$k_\rho = \frac{\chi}{\eta}\frac{\partial T}{\partial \rho}; \quad k_p = \frac{\chi}{\eta}\frac{\partial T}{\partial p}; \quad k_{\alpha_1} = \frac{\chi}{\eta}\frac{\partial T}{\partial \alpha_1}; \quad k_{\rho_1^0} = \frac{\chi}{\eta}\frac{\partial T}{\partial \rho_1^0}, \ldots;$$

$$k_{\alpha_{m-1}} = \frac{\chi}{\eta}\frac{\partial T}{\partial \alpha_{m-1}}; \quad k_{\rho_{m-1}^0} = \frac{\chi}{\eta}\frac{\partial T}{\partial \rho_{m-1}^0}; \quad k_{\alpha_{m+1}} = \frac{\chi}{\eta}\frac{\partial T}{\partial \alpha_{m+1}}; \quad k_{\alpha_n} = \frac{\chi}{\eta}\frac{\partial T}{\partial \alpha_n}.$$

The corresponding expressions for H, G_i, and for the adiabatic velocity of sound c have the form

$$H = \left[\frac{\partial \varepsilon}{\partial p} + \sum_{i=1}^{m-1}\frac{\partial \varepsilon_i}{\partial p}\left(\frac{\partial \varepsilon_i}{\partial \rho_i^0}\right)^{-1}\left(\frac{\alpha_i}{\rho_i^0}\frac{\partial \varepsilon}{\partial \alpha_i} - \frac{\partial \varepsilon}{\partial \rho_i^0}\right)\right]^{-1}, \quad G_i = \frac{1}{\rho_i^0}\left(\frac{\partial \varepsilon_i}{\partial \rho_i^0}\right)^{-1}\left(\frac{p}{\rho_i^0} - \rho c^2 \frac{\partial \varepsilon_i}{\partial p}\right),$$

$$c = \sqrt{\frac{\dfrac{p}{\rho} - \rho\dfrac{\partial \varepsilon}{\partial \rho} - \sum\limits_{i=1}^{m-1}\left[\dfrac{p}{\rho_i^0}\dfrac{\partial \varepsilon}{\partial \rho_i^0}\left(\dfrac{\partial \varepsilon_i}{\partial \rho_i^0}\right)^{-1} + \alpha_i\dfrac{\partial \varepsilon}{\partial \alpha_i}\left(1 - p\left((\rho_i^0)^2\dfrac{\partial \varepsilon_i}{\partial \rho_i^0}\right)^{-1}\right)\right] - \sum\limits_{j=m+1}^{n}\alpha_j\dfrac{\partial \varepsilon}{\partial \alpha_j}}{\rho\left[\dfrac{\partial \varepsilon}{\partial p} + \sum\limits_{i=1}^{m-1}\dfrac{\partial \varepsilon_i}{\partial p}\left(\dfrac{\partial \varepsilon_i}{\partial \rho_i^0}\right)^{-1}\left(\dfrac{\alpha_i}{\rho_i^0}\dfrac{\partial \varepsilon}{\partial \alpha_i} - \dfrac{\partial \varepsilon}{\partial \rho_i^0}\right)\right]}}.$$

If in describing the behavior of compressible components of the mixture one uses the equation of state

$$\varepsilon_i = \frac{p - c_{*i}^2(\rho_i^0 - \rho_{*i})}{\rho_i^0(\gamma_i - 1)}, \tag{8}$$

where γ_i, ρ_{*i}, and c_{*i} are the constants of the ith fraction that determine its individual properties, then for an n-component mixture with the first m compressible fractions Eq. (3) takes the form

$$\varepsilon = \frac{1}{\rho}\left[\sum_{i=1}^{m-1}(pB_{im} - d_{im}\rho_i^0 + b_{im})\,\alpha_i + pB_m + b_m + \sum_{j=m+1}^{n}\alpha_j\rho_j^0\varepsilon_j\right] - d_m. \tag{9}$$

Here $B_i = 1/(\gamma_i - 1)$, $B_{im} = B_i - B_m$, $d_i = c_{*i}^2 B_i$, $d_{im} = d_i - d_m$, $b_i = d_i\rho_{*i}$, $b_{im} = b_i - b_m$. The equation of state (8) for the ith fraction can be rewritten as

$$\varepsilon_i = \frac{pB_i + b_i}{\rho_i^0} - d_i. \tag{10}$$

The corresponding expressions for H, G_i, and c take the form

$$H = \rho\left[B_m + \sum_{i=1}^{m-1}\frac{\alpha_i(b_mB_i - b_iB_m)}{b_i + pB_i}\right]^{-1}, \quad G_i = \frac{\rho c^2 B_i - p}{b_i + pB_i}, \quad c = \sqrt{\frac{b_m + p\left[1 + B_m - \sum\limits_{i=1}^{m-1}\dfrac{\alpha_i(b_{im} + pB_{im})}{b_i + pB_i}\right]}{\rho\left[B_m + \sum\limits_{i=1}^{m-1}\dfrac{\alpha_i(b_mB_i - b_iB_m)}{b_i + pB_i}\right]}}. \tag{11}$$

For a mixture of ideal gases the relations for H, G_i, and c are simplified still further:

$$H = \frac{\rho}{B_m}, \quad G_i = \frac{\rho c^2 B_i - p}{p B_i}, \quad c = \sqrt{\frac{p}{\rho B_m}\left(1 + B_m - \sum_{i=1}^{m-1} \frac{\alpha_i B_{im}}{B_i}\right)} . \tag{12}$$

We will rewrite the system of equations (6) in a vector-matrix form:

$$\frac{\partial \mathbf{U}}{\partial t} + \mathrm{A}\,\frac{\partial \mathbf{U}}{\partial x} = \mathbf{S}, \tag{13}$$

where

$$\mathbf{U} = (\rho, u, p, \rho_1^0, \alpha_1, ..., \rho_{m-1}^0, \alpha_{m-1}, \alpha_{m+1}, ..., \alpha_n; W)^{\mathrm{Tr}}, \quad \mathbf{S} = (0, ..., 0, -W/\eta)^{\mathrm{Tr}};$$

$$\mathrm{A} = \begin{pmatrix}
u & \rho & 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & u & 1/\rho & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & \rho c^2 & u & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & H \\
0 & \rho_1^0 G_1 & 0 & u & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & \alpha_1 (1 - G_1) & 0 & 0 & u & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
0 & \rho_{m-1}^0 G_{m-1} & 0 & 0 & 0 & \dots & u & 0 & 0 & \dots & 0 & 0 \\
0 & \alpha_{m-1}(1 - G_{m-1}) & 0 & 0 & 0 & \dots & 0 & u & 0 & \dots & 0 & 0 \\
0 & \alpha_{m+1} & 0 & 0 & 0 & \dots & 0 & 0 & u & \dots & 0 & 0 \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
0 & \alpha_n & 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & u & 0 \\
k_\rho & 0 & k_p & k_{\rho_1^0} & k_{\alpha_1} & \dots & k_{\rho_{m-1}^0} & k_{\alpha_{m-1}} & k_{\alpha_{m+1}} & \dots & k_{\alpha_n} & u
\end{pmatrix}.$$

The characteristic equation of system (6) is written as

$$(\xi - (u - c_1))\,(\xi - (u - c_2))\,(\xi - u)^{n+m-2}\,(\xi - (u + c_2))\,(\xi - (u + c_1)) = 0, \tag{14}$$

where $\xi = dx/dt$. The expressions for the velocities c_1 and c_2 have the form

$$c_1 = \sqrt{\frac{1}{2}\left\{c^2 + k_p H + \sqrt{c^4 + H\left[k_p\,(2c^2 + k_p H) + 4\left(k_\rho + \frac{1}{\rho}\left(\sum_{i=1}^{m-1}(k_{\rho_i^0}\rho_i^0 G_i + k_{\alpha_i}\alpha_i\,(1 - G_i)) + \sum_{j=m+1}^{n} k_{\alpha_j}\alpha_j\right)\right)\right]}\right\}},$$

$$c_2 = \sqrt{\frac{1}{2}\left\{c^2 + k_p H - \sqrt{c^4 + H\left[k_p\,(2c^2 + k_p H) + 4\left(k_\rho + \frac{1}{\rho}\left(\sum_{i=1}^{m-1}(k_{\rho_i^0}\rho_i^0 G_i + k_{\alpha_i}\alpha_i\,(1 - G_i)) + \sum_{j=m+1}^{n} k_{\alpha_j}\alpha_j\right)\right)\right]}\right\}}.$$

The roots of the characteristic equation (14) are real numbers; moreover, the matrix A can be presented as

$$\mathrm{A} = \Omega^{-1}\Lambda\Omega, \tag{15}$$

where

$$
\Omega = \left(\begin{array}{cccccccccccc}
1 & \frac{\rho c_1}{k_\rho}\left(k_p - \frac{c_1^2}{H}\right) & \frac{c_1^2}{k_\rho H} & \frac{k_{\rho_1^0}}{k_\rho} & \frac{k_{\alpha_1}}{k_\rho} & \dots & \frac{k_{\rho_{m-1}^0}}{k_\rho} & \frac{k_{\alpha_{m-1}}}{k_\rho} & \frac{k_{\alpha_{m+1}}}{k_\rho} & \dots & \frac{k_{\alpha_n}}{k_\rho} & -\frac{c_1}{k_\rho} \\
1 & \frac{\rho c_2}{k_\rho}\left(k_p - \frac{c_2^2}{H}\right) & \frac{c_2^2}{k_\rho H} & \frac{k_{\rho_1^0}}{k_\rho} & \frac{k_{\alpha_1}}{k_\rho} & \dots & \frac{k_{\rho_{m-1}^0}}{k_\rho} & \frac{k_{\alpha_{m-1}}}{k_\rho} & \frac{k_{\alpha_{m+1}}}{k_\rho} & \dots & \frac{k_{\alpha_n}}{k_\rho} & -\frac{c_2}{k_\rho} \\
1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & -\frac{\rho}{\alpha_n} & 0 \\
\multicolumn{12}{c}{\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots} \\
1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & -\frac{\rho}{\alpha_{m+1}} & \dots & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & \dots & 0 & -\frac{\rho}{\alpha_{m-1}(1-G_{m-1})} & 0 & \dots & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & \dots & -\frac{\rho}{\rho_{m-1}^0 G_{m-1}} & 0 & 0 & \dots & 0 & 0 \\
\multicolumn{12}{c}{\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots} \\
1 & 0 & 0 & 0 & -\frac{\rho}{\alpha_1(1-G_1)} & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\
1 & 0 & 0 & -\frac{\rho}{\rho_1^0 G_1} & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\
1 & -\frac{\rho c_2}{k_\rho}\left(k_p - \frac{c_2^2}{H}\right) & \frac{c_2^2}{k_\rho H} & \frac{k_{\rho_1^0}}{k_\rho} & \frac{k_{\alpha_1}}{k_\rho} & \dots & \frac{k_{\rho_{m-1}^0}}{k_\rho} & \frac{k_{\alpha_{m-1}}}{k_\rho} & \frac{k_{\alpha_{m+1}}}{k_\rho} & \dots & \frac{k_{\alpha_n}}{k_\rho} & \frac{c_2}{k_\rho} \\
1 & -\frac{\rho c_1}{k_\rho}\left(k_p - \frac{c_1^2}{H}\right) & \frac{c_1^2}{k_\rho H} & \frac{k_{\rho_1^0}}{k_\rho} & \frac{k_{\alpha_1}}{k_\rho} & \dots & \frac{k_{\rho_{m-1}^0}}{k_\rho} & \frac{k_{\alpha_{m-1}}}{k_\rho} & \frac{k_{\alpha_{m+1}}}{k_\rho} & \dots & \frac{k_{\alpha_n}}{k_\rho} & \frac{c_1}{k_\rho}
\end{array} \right) ;
$$

$\Lambda = (\lambda_p \delta_{pk})$, λ_p are the eigenvalues of the matrix A and δ_{pk} is the Kronecker symbol, therefore system (6) is hyperbolic but not reducible to a divergent form.

We will consider in more detail a three-component mixture each component of which is compressible. In this case the system of equations (6) takes the form

$$
\begin{gathered}
\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} = 0\,, \quad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0\,, \quad \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + \rho c^2\frac{\partial u}{\partial x} + H\frac{\partial W}{\partial x} = 0\,, \\
\frac{\partial \rho_1^0}{\partial t} + u\frac{\partial \rho_1^0}{\partial x} + \rho_1^0 G_1\frac{\partial u}{\partial x} = 0\,, \quad \frac{\partial \alpha_1}{\partial t} + u\frac{\partial \alpha_1}{\partial x} + \alpha_1(1-G_1)\frac{\partial u}{\partial x} = 0\,, \\
\frac{\partial \rho_2^0}{\partial t} + u\frac{\partial \rho_2^0}{\partial x} + \rho_2^0 G_2\frac{\partial u}{\partial x} = 0\,, \quad \frac{\partial \alpha_2}{\partial t} + u\frac{\partial \alpha_2}{\partial x} + \alpha_2(1-G_2)\frac{\partial u}{\partial x} = 0\,, \\
\frac{\partial W}{\partial t} + u\frac{\partial W}{\partial x} + k_\rho\frac{\partial \rho}{\partial x} + k_p\frac{\partial p}{\partial x} + k_{\rho_1^0}\frac{\partial \rho_1^0}{\partial x} + k_{\alpha_1}\frac{\partial \alpha_1}{\partial x} + k_{\rho_2^0}\frac{\partial \rho_2^0}{\partial x} + k_{\alpha_2}\frac{\partial \alpha_2}{\partial x} + \frac{1}{\eta}W = 0\,,
\end{gathered}
\tag{16}
$$

where

$$k_\rho = \frac{\chi}{\eta}\frac{\partial T}{\partial \rho};\quad k_p = \frac{\chi}{\eta}\frac{\partial T}{\partial p};\quad k_{\rho_1^0} = \frac{\chi}{\eta}\frac{\partial T}{\partial \rho_1^0};\quad k_{\alpha_1} = \frac{\chi}{\eta}\frac{\partial T}{\partial \alpha_1};\quad k_{\rho_2^0} = \frac{\chi}{\eta}\frac{\partial T}{\partial \rho_2^0};\quad k_{\alpha_2} = \frac{\chi}{\eta}\frac{\partial T}{\partial \alpha_2}.$$

We will represent system (16) in a vector-matrix form of (13), in which

$$\mathbf{U} = \begin{pmatrix} \rho \\ u \\ p \\ \rho_1^0 \\ \alpha_1 \\ \rho_2^0 \\ \alpha_2 \\ W \end{pmatrix},\quad \mathrm{A} = \begin{pmatrix} u & \rho & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & u & 1/\rho & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho c^2 & u & 0 & 0 & 0 & 0 & H \\ 0 & \rho_1^0 G_1 & 0 & u & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 (1 - G_1) & 0 & 0 & u & 0 & 0 & 0 \\ 0 & \rho_2^0 G_2 & 0 & 0 & 0 & u & 0 & 0 \\ 0 & \alpha_2 (1 - G_2) & 0 & 0 & 0 & 0 & u & 0 \\ k_\rho & 0 & k_p & k_{\rho_1^0} & k_{\alpha_1} & k_{\rho_2^0} & k_{\alpha_2} & u \end{pmatrix},\quad \mathbf{S} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -W/\eta \end{pmatrix}. \tag{17}$$

The matrix A has eight real eigenvalues:

$$u \pm c_1, u, u, u, u, u \pm c_2,$$

where

$$c_1 = \sqrt{\frac{1}{2}\left\{c^2 + k_p H + \sqrt{c^4 + H\left[k_p (2c^2 + k_p H) + 4\left(k_\rho + \frac{1}{\rho}(k_{\rho_1^0}\rho_1^0 G_1 + k_{\alpha_1}\alpha_1 (1 - G_1) + k_{\rho_2^0}\rho_2^0 G_2 + k_{\alpha_2}\alpha_2 (1 - G_2))\right)\right]}\right\}},$$

$$c_2 = \sqrt{\frac{1}{2}\left\{c^2 + k_p H - \sqrt{c^4 + H\left[k_p (2c^2 + k_p H) + 4\left(k_\rho + \frac{1}{\rho}(k_{\rho_1^0}\rho_1^0 G_1 + k_{\alpha_1}\alpha_1 (1 - G_1) + k_{\rho_2^0}\rho_2^0 G_2 + k_{\alpha_2}\alpha_2 (1 - G_2))\right)\right]}\right\}}. \tag{18}$$

Note that c_1 determines the velocity of propagation of gas-dynamical disturbances, and c_2 of thermal ones. The corresponding matrices Ω, Λ, and Ω^{-1} in the representation of (15) have the form

$$\Omega = \begin{pmatrix} 1 & e_1 & f_1 & r_1 & q_1 & r_2 & q_2 & s_1 \\ 1 & e_2 & f_2 & r_1 & q_1 & r_2 & q_2 & s_2 \\ 1 & 0 & 0 & 0 & 0 & 0 & g_2 & 0 \\ 1 & 0 & 0 & 0 & 0 & h_2 & 0 & 0 \\ 1 & 0 & 0 & 0 & g_1 & 0 & 0 & 0 \\ 1 & 0 & 0 & h_1 & 0 & 0 & 0 & 0 \\ 1 & -e_2 & f_2 & r_1 & q_1 & r_2 & q_2 & -s_2 \\ 1 & -e_1 & f_1 & r_1 & q_1 & r_2 & q_2 & -s_1 \end{pmatrix},\quad \Lambda = \begin{pmatrix} u - c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & u - c_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & u & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & u & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & u & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & u & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & u + c_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & u + c_1 \end{pmatrix},$$

$$\Omega^{-1} = \frac{1}{2g} \tag{19}$$

$$
\times \begin{pmatrix}
\frac{g_1 g_2 h_1 h_2 f_2}{f_1 - f_2} & -\frac{g_1 g_2 h_1 h_2 f_1}{f_1 - f_2} & 2g_1 q_2 h_1 h_2 & 2g_1 g_2 h_1 r_2 & 2q_1 g_2 h_1 h_2 & 2g_1 g_2 r_1 h_2 & -\frac{g_1 g_2 h_1 h_2 f_1}{f_1 - f_2} & \frac{g_1 g_2 h_1 h_2 f_2}{f_1 - f_2} \\
\frac{s_2 g}{e_1 s_2 - e_2 s_1} & -\frac{s_1 g}{e_1 s_2 - e_2 s_1} & 0 & 0 & 0 & 0 & \frac{s_1 g}{e_1 s_2 - e_2 s_1} & -\frac{s_2 g}{e_1 s_2 - e_2 s_1} \\
\frac{g}{f_1 - f_2} & -\frac{g}{f_1 - f_2} & 0 & 0 & 0 & 0 & -\frac{g}{f_1 - f_2} & \frac{g}{f_1 - f_2} \\
-\frac{g_1 g_2 h_2 f_2}{f_1 - f_2} & \frac{g_1 g_2 h_2 f_1}{f_1 - f_2} & -2g_1 q_2 h_2 & -2g_1 g_2 r_2 & -2q_1 g_2 h_2 & 2F_1 & \frac{g_1 g_2 h_2 f_1}{f_1 - f_2} & -\frac{g_1 g_2 h_2 f_2}{f_1 - f_2} \\
-\frac{g_2 h_1 h_2 f_2}{f_1 - f_2} & \frac{g_2 h_1 h_2 f_1}{f_1 - f_2} & -2q_2 h_1 h_2 & -2g_2 h_1 r_2 & 2F_2 & -2r_1 g_2 h_2 & \frac{g_2 h_1 h_2 f_1}{f_1 - f_2} & -\frac{g_2 h_1 h_2 f_2}{f_1 - f_2} \\
-\frac{g_1 g_2 h_1 f_2}{f_1 - f_2} & \frac{g_1 g_2 h_1 f_1}{f_1 - f_2} & -2g_1 q_2 h_1 & 2F_3 & -2q_1 g_2 h_1 & -2g_1 g_2 r_1 & \frac{g_1 g_2 h_1 f_1}{f_1 - f_2} & -\frac{g_1 g_2 h_1 f_2}{f_1 - f_2} \\
-\frac{g_1 h_1 h_2 f_2}{f_1 - f_2} & \frac{g_1 h_1 h_2 f_1}{f_1 - f_2} & 2F_4 & -2g_1 h_1 r_2 & -2q_1 h_1 h_2 & -2g_1 r_1 h_2 & \frac{g_1 h_1 h_2 f_1}{f_1 - f_2} & -\frac{g_1 h_1 h_2 f_2}{f_1 - f_2} \\
-\frac{e_2 g}{e_1 s_2 - e_2 s_1} & \frac{e_1 g}{e_1 s_2 - e_2 s_1} & 0 & 0 & 0 & 0 & -\frac{e_1 g}{e_1 s_2 - e_2 s_1} & \frac{e_2 g}{e_1 s_2 - e_2 s_1}
\end{pmatrix},
$$

where

$$e_1 = \frac{\rho c_1}{k_\rho}\left(k_p - \frac{c_1^2}{H}\right); \quad e_2 = \frac{\rho c_2}{k_\rho}\left(k_p - \frac{c_2^2}{H}\right); \quad f_1 = \frac{c_1^2}{k_\rho H}; \quad f_2 = \frac{c_2^2}{k_\rho H};$$

$$g_1 = -\frac{\rho}{\alpha_1 (1 - G_1)}; \quad g_2 = -\frac{\rho}{\alpha_2 (1 - G_2)};$$

$$h_1 = -\frac{\rho}{\rho_1^0 G_1}; \quad h_2 = -\frac{\rho}{\rho_2^0 G_2}; \quad r_1 = \frac{k_{\rho_1^0}}{k_\rho}; \quad r_2 = \frac{k_{\rho_2^0}}{k_\rho}; \quad q_1 = \frac{k_{\alpha_1}}{k_\rho}; \quad q_2 = \frac{k_{\alpha_2}}{k_\rho}; \quad s_1 = -\frac{c_1}{k_\rho}; \quad s_2 = -\frac{c_2}{k_\rho};$$

$$g = q_1 g_2 h_1 h_2 + g_1 q_2 h_1 h_2 + g_1 g_2 r_1 h_2 + g_1 g_2 h_1 r_2 - g_1 g_2 h_1 h_2;$$

$$F_1 = g_1 q_2 h_2 - g_1 g_2 h_2 + q_1 g_2 h_2 + g_1 g_2 r_2; \quad F_2 = q_2 h_1 h_2 - g_2 h_1 h_2 + g_2 r_1 h_2 + g_2 h_1 r_2;$$

$$F_3 = g_1 q_2 h_1 - g_1 g_2 h_1 + g_1 g_2 r_1 + q_1 g_2 h_1; \quad F_4 = q_1 h_1 h_2 - g_1 h_1 h_2 + g_1 r_1 h_2 + g_1 h_1 r_2.$$

For a mixture of ideal gases the expression for the average temperature (4) yields

$$T = p\left[\frac{\alpha_1}{\rho_1^0 R_1} + \frac{\alpha_2}{\rho_2^0 R_2} + \frac{(1 - \alpha_1 - \alpha_2)^2}{(\rho - \alpha_1 \rho_1^0 - \alpha_2 \rho_2^0) R_3}\right], \tag{20}$$

where R_i is the gas constant of the ith fraction. With the use of Eq. (20) the coefficients k_ρ, k_p, $k_{\rho_1^0}$, k_{α_1}, $k_{\rho_2^0}$, and k_{α_2} entering into Eq. (18) take the form

$$k_\rho = -\frac{\chi}{\eta}\frac{(1 - \alpha_1 - \alpha_2)^2 p}{(\rho - \alpha_1 \rho_1^0 - \alpha_2 \rho_2^0)^2 R_3}, \quad k_p = \frac{\chi}{\eta}\left[\frac{\alpha_1}{\rho_1^0 R_1} + \frac{\alpha_2}{\rho_2^0 R_2} + \frac{(1 - \alpha_1 - \alpha_2)^2}{(\rho - \alpha_1 \rho_1^0 - \alpha_2 \rho_2^0) R_3}\right],$$

$$k_{\rho_1^0} = -\frac{\alpha_1 \chi p}{\eta}\left[\frac{1}{(\rho_1^0)^2 R_1} - \frac{(1-\alpha_1-\alpha_2)^2}{(\rho - \alpha_1\rho_1^0 - \alpha_2\rho_2^0)^2 R_3}\right],$$

$$k_{\alpha_1} = \frac{\chi p}{\eta}\left[\frac{1}{\rho_1^0 R_1} + \frac{(1-\alpha_1-\alpha_2)\left[\alpha_1\rho_1^0 - 2(\rho-\alpha_2\rho_2^0) + (1-\alpha_2)\rho_1^0\right]}{(\rho - \alpha_1\rho_1^0 - \alpha_2\rho_2^0)^2 R_3}\right],$$

$$k_{\rho_2^0} = -\frac{\alpha_2 \chi p}{\eta}\left[\frac{1}{(\rho_2^0)^2 R_2} - \frac{(1-\alpha_1-\alpha_2)^2}{(\rho - \alpha_1\rho_1^0 - \alpha_2\rho_2^0)^2 R_3}\right],$$

$$k_{\alpha_2} = \frac{\chi p}{\eta}\left[\frac{1}{\rho_2^0 R_2} + \frac{(1-\alpha_1-\alpha_2)\left[\alpha_2\rho_2^0 - 2(\rho-\alpha_1\rho_1^0) + (1-\alpha_1)\rho_2^0\right]}{(\rho - \alpha_1\rho_1^0 - \alpha_2\rho_2^0)^2 R_3}\right].$$

In particular, for a mixture consisting of nitrogen ($\alpha_1 = 0.781$, $\gamma_1 = 1.524$, $\rho_1^0 = 1.149$ kg/m^3), oxygen ($\alpha_2 = 0.21$, $\gamma_2 = 1.409$, $\rho_2^0 = 1.314$ kg/m^3), and argon ($\alpha_3 = 0.009$, $\gamma_3 = 1.838$, $\rho_3^0 = 1.640$ kg/m^3) under normal conditions, the values of velocities c_1, c_2, and c are equal to 340.85, 0.664, and 340.86 m/sec, respectively. In calculations the thermal conductivity coefficient of the mixture was determined from the expression

$$\chi = \frac{1}{\rho}(\rho_1\chi_1 + \rho_2\chi_2 + \rho_3\chi_3), \tag{21}$$

where $\chi_1 = 2.57 \cdot 10^{-2}$ kg·m/(sec^3·K) for nitrogen, $\chi_2 = 2.47 \cdot 10^{-2}$ kg·m/(sec^3·K) for oxygen, and $\chi_3 = 1.77 \cdot 10^{-2}$ kg·m/(sec^3·K) for argon. The coefficient $\eta = 10^{-4}$ sec.

The considered model of a heat-conducting heterogeneous medium is the one-parameter model with the sole unknown quantity — the coefficient of thermal relaxation η. Note that if there are experimental data on propagation of, say, thermal waves in a mixture, then from them we can determine the parameter η, which entirely verifies the model.

Numerical Integration of Model's Equations. We will consider a one-dimensional plane flow of a three-component mixture consisting of ideal gases. The average temperature of the mixture was calculated from Eq. (20).

For numerical integration of system (16) with the vectors **U**, **S** and matrix A from (17), we use the Courant–Isaacson–Rees finite-difference scheme (see [5]):

$$\frac{\mathbf{U}_i^{k+1} - \mathbf{U}_i^k}{\Delta t} + \mathrm{A}_i^k \frac{\mathbf{U}_{i+1/2}^k - \mathbf{U}_{i-1/2}^k}{\Delta x} = \mathbf{S}_i^k, \tag{22}$$

where

$$\mathbf{U}_{m+1/2}^k = \frac{1}{2}\left(\mathbf{U}_m^k + \mathbf{U}_{m+1}^k\right) + \frac{1}{2}\left\{\Omega^{-1}\,[\mathrm{sign}\ (\Lambda)]\,\Omega\right\}_m^k \left(\mathbf{U}_m^k - \mathbf{U}_{m+1}^k\right), \quad m = i, i-1.$$

The matrices Ω^{-1}, Λ, and Ω were determined according to Eqs. (19).

In calculating flows of a nonconductive mixture, the following procedure was used. Since the mixture of ideal gases is considered for which $b_1 = 0$ and $b_2 = 0$, the equations of the heat influx for these fractions, considered together with the continuity equation for the mixture as a whole, are reduced to a divergent form:

$$\frac{\partial \rho\varphi_1}{\partial t} + \frac{\partial \rho\varphi_1 u}{\partial x} = 0, \quad \frac{\partial \rho\varphi_2}{\partial t} + \frac{\partial \rho\varphi_2 u}{\partial x} = 0,$$

where

$$\varphi_1 = \ln\left(\frac{1}{\rho_1}\left(\frac{p}{\rho_1^0}\right)^{B_1}\right); \quad \varphi_2 = \ln\left(\frac{1}{\rho_2}\left(\frac{p}{\rho_2^0}\right)^{B_2}\right).$$

Thus, if we introduce the vector $\mathbf{V} = (\rho,\ \rho u,\ \rho e,\ \alpha_1\rho_1^0,\ \rho\varphi_1,\ \alpha_2\rho_2^0,\ \rho\varphi_2)^{\mathrm{Tr}}$, then the determining system of equations (13) with the parameters

$$\mathbf{U} = \begin{pmatrix} \rho \\ u \\ p \\ \rho_1^0 \\ \alpha_1 \\ \rho_2^0 \\ \alpha_2 \end{pmatrix}, \quad \mathrm{A} = \begin{pmatrix} u & \rho & 0 & 0 & 0 & 0 & 0 \\ 0 & u & 1/\rho & 0 & 0 & 0 & 0 \\ 0 & \rho c^2 & u & 0 & 0 & 0 & 0 \\ 0 & \rho_1^0 G_1 & 0 & u & 0 & 0 & 0 \\ 0 & \alpha_1(1-G_1) & 0 & 0 & u & 0 & 0 \\ 0 & \rho_2^0 G_2 & 0 & 0 & 0 & u & 0 \\ 0 & \alpha_2(1-G_1) & 0 & 0 & 0 & 0 & u \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

subject to the equation of state (9) that in the considered case takes the form

$$\varepsilon = \frac{p}{\rho}(\alpha_1 B_{13} + \alpha_2 B_{23} + B_3),$$

is written as

$$\frac{\partial \mathbf{V}}{\partial t} + \frac{\partial \Pi(\mathbf{V})}{\partial x} = 0, \tag{23}$$

where

$$\Pi(\mathbf{V}) = (\rho u,\ p + \rho u^2;\ (p + \rho e)\, u,\ \rho_1 u,\ \rho\varphi_1 u,\ \rho_2 u,\ \rho\varphi_2 u)^{\mathrm{Tr}}.$$

Transition from the "primary" variables U_i to the "new" variables V_i is made in the following way:

$$V_1 \equiv \rho = U_1, \quad V_2 \equiv \rho u = U_1 U_2, \quad V_3 \equiv \rho\left(\varepsilon + \frac{u^2}{2}\right) = U_3(B_{13}U_5 + B_{23}U_7 + B_3) + \frac{U_1 U_2^2}{2},$$

$$V_4 \equiv \rho_1^0 \alpha_1 = U_4 U_5, \quad V_5 \equiv \rho \ln\left(\frac{1}{\alpha_1\rho_1^0}\left(\frac{p}{\rho_1^0}\right)^{B_1}\right) = U_1 \ln\left(\frac{1}{U_5 U_4}\left(\frac{U_3}{U_4}\right)^{B_1}\right),$$

$$V_6 \equiv \rho_2^0 \alpha_2 = U_6 U_7, \quad V_7 \equiv \rho \ln\left(\frac{1}{\alpha_2\rho_2^0}\left(\frac{p}{\rho_2^0}\right)^{B_2}\right) = U_1 \ln\left(\frac{1}{U_7 U_6}\left(\frac{U_3}{U_6}\right)^{B_2}\right).$$

The inverse transformations are performed with the aid of the expressions

$$U_1 \equiv \rho = V_1, \quad U_2 \equiv u = \frac{V_2}{V_1}, \quad U_3 \equiv p = \frac{1}{B_3}\left(V_3 - \frac{V_2^2}{2V_1} - A_1 B_{13} V_4 - A_2 B_{23} V_6\right),$$

$$U_4 \equiv \rho_1^0 = \frac{1}{A_1 B_3}\left(V_3 - \frac{V_2^2}{2V_1} - A_1 B_{13} V_4 - A_2 B_{23} V_6\right), \quad U_5 \equiv \alpha_1 = A_1 B_3 V_4 \left(V_3 - \frac{V_2^2}{2V_1} - A_1 B_{13} V_4 - A_2 B_{23} V_6\right)^{-1},$$

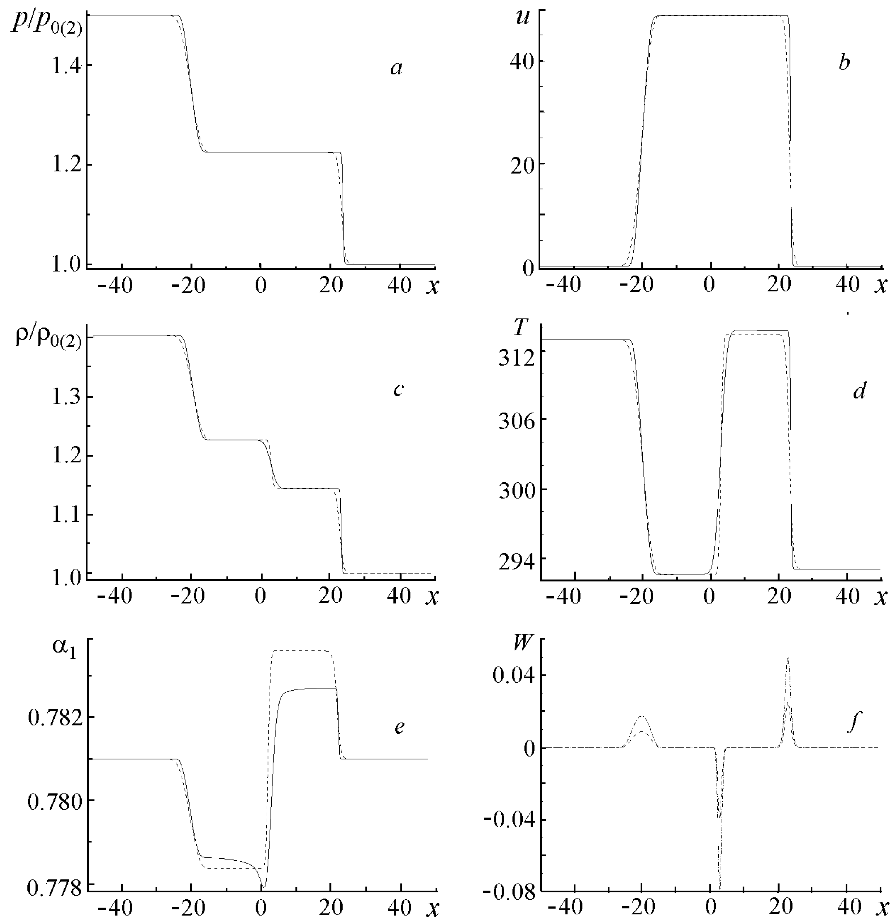

Fig. 1. Distribution of the parameters of flow along x: in a heat-conducting mixture — dashed curves; in a mixture with doubled thermal conductivity coefficients — dashed-dotted curves; for a nonconducting mixture — solid curves.

$$U_6 \equiv \rho_2^0 = \frac{1}{A_2 B_3}\left(V_3 - \frac{V_2^2}{2V_1} - A_1 B_{13} V_4 - A_2 B_{23} V_6\right), \quad U_7 \equiv \alpha_2 = A_2 B_3 V_6 \left(V_3 - \frac{V_2^2}{2V_1} - A_1 B_{13} V_4 - A_2 B_{23} V_6\right)^{-1},$$

where

$$A_1 = \left[V_4 \exp\left(\frac{V_5}{V_1}\right)\right]^{\frac{1}{B_1}}; \quad A_2 = \left[V_6 \exp\left(\frac{V_7}{V_1}\right)\right]^{\frac{1}{B_2}}.$$

The finite-volume scheme for the system in a divergent form (23) has the form

$$\frac{\mathbf{V}_i^{k+1} - \mathbf{V}_i^k}{\Delta t} + \frac{\Pi_{i+1/2}^k - \Pi_{i-1/2}^k}{\Delta x} = 0\,. \tag{24}$$

In calculating the fluxes through the faces of the cells the Harten–Lax–Van Leer approach (see [5]) was used:

$$\Pi_{l-1/2}^{k}=\begin{cases}\Pi_{l-1}^{k}\,, & \text{if}\quad \lambda_1>0\,,\\ \dfrac{\lambda_7\Pi_{l-1}^{k}-\lambda_1\Pi_{l}^{k}+\lambda_1\lambda_7\,(\mathbf{V}_l^k-\mathbf{V}_{l-1}^k)}{\lambda_7-\lambda_1}\,, & \text{if}\quad \lambda_1<0\,,\ \ \lambda_7>0\,,\\ \Pi_{l}^{k}\,, & \text{if}\quad \lambda_7<0\,.\end{cases}$$

With the used of the counterflow scheme (22) the Riemann problem in a heat-conducting mixture consisting of nitrogen, oxygen, and argon was solved numerically. The parameters of the mixture prior to decomposition were: to the left of the diaphragm ($x<0$) $p_{0(1)}$ = 0.15 MPa, $u_{0(1)}$ = 0, $\alpha_{10(1)}$ = 0.781, $T_{10(1)}$ = 313 K, $\rho^0_{10(1)}$ = 1.614 kg/m^3, $\gamma_{1(1)}$ = 1.524, $\chi_{1(1)}$ = 2.57·10^{-2} kg·m/(sec^3·K), $\alpha_{20(1)}$ = 0.21, $T_{20(1)}$ = 313 K, $\rho^0_{20(1)}$ = 1.845 kg/m^3, $\gamma_{2(1)}$ = 1.409, $\gamma_{2(1)}$ = 2.47·10^{-2} kg·m/(sec^3·K), $\alpha_{30(1)}$ = 0.009, $T_{30(1)}$ = 313 K, $\rho^0_{30(1)}$ = 2.303 kg/m^3, $\gamma_{3(1)}$ = 1.838, $\chi_{3(1)}$ = 1.77·10^{-2} kg·m/(sec^3·K), $\eta_{(1)}$ = 0.05 sec; to the right of the diaphragm ($x>0$) $p_{0(2)}$ = 0.1 MPa, $u_{0(2)}$ = 0, $\alpha_{20(1)}$ = 0.781, $T_{10(2)}$ = 293 K, $\rho^0_{10(2)}$ = 1.149 kg/m^3, $\gamma_{1(2)}$ = 1.524, $\chi_{1(2)}$ = 2.57·10^{-2} kg·m/(sec^3·K), $\alpha_{20(2)}$ = 0.21, $T_{20(2)}$ = 293 K, $\rho^0_{20(2)}$ = 1.314 kg/m^3, $\gamma_{2(2)}$ = 1.409, $\chi_{2(2)}$ = 2.47·10^{-2} kg·m/(sec^3·K), $\alpha_{3(2)}$ = 0.009, $T_{30(2)}$ = 293 K, $\rho^0_{30(2)}$ = 1.64 kg/m^3, $\gamma_{3(2)}$ = 1.838, $\chi_{3(2)}$ = 1.77·10^{-2} kg·m/(sec^3·K), $\eta_{(2)}$ = 0.05 sec. At time t = 0 the diaphragm is instantly removed, and the regime of flow with a shock wave moving to the right and rarefaction wave moving to the left is realized.

Figure 1 presents the results of computations of the decomposition of arbitrary discontinuity in both a heat-conducting and nonconducting mixture that were obtained with the use of schemes (22) and (24) by the time t = 0.06 sec. The computations were performed on a dimensional grid consisting of 1000 meshes. From Fig. 1f it is seen that the most intense heat fluxes are observed near the contact boundary, and the less intense ones in the shock and rarefaction waves. On twofold increase in the thermal conductivity coefficient of each of the mixture components, the heat fluxes near the contact boundary, shock wave, and rarefaction wave increase, the remaining flow parameters practically remaining intact. It should also be noted that with the use of scheme (22) the position of the contact boundary is determined more precisely as against the Harten–Lax–Van der Leer method (see Fig. 1c).

Conclusions. The application of the generalized Fourier heat-transfer law with heat flux relaxation ensures the hyperbolicity of the equations in the model of one-velocity multicomponent heat-conducting medium, which, in turn, makes it possible to obtain a physically consistent pattern of flow and, moreover, allows one to use the well-established numerical methods of solving hyperbolic systems of equations.

NOTATION

c, adiabatic velocity of sound in a mixture; c_{*i}, constant of equation of state; $e = \varepsilon + \frac{1}{2}u^2$, specific full energy of mixture; $\mathbf{F}$, mass force density; J_{ij}, intensity of conversion of mass from ith fraction into jth one per unit volume of mixture; p, pressure; Q_{ij}, heat release per unit time per unit volume of mixture arising as a result of conversion of ith fraction into jth one; R_{ij}, quantity of heat per unit time per unit volume of mixture entering into ith fraction from jth one by radiation; t, time; T, averaged temperature; Tr, transposition operator; $\mathbf{u}$, velocity vector; $\mathbf{W}_i$, vector of heat flux density for ith fraction; $\mathbf{W} = \sum_{i=1}^{n} \alpha_i \mathbf{W}_i$, average vector of heat flux; x, spatial variable; α_i, volumetric fraction; γ_i, constant of equation of state; ε_i, specific internal energy; $\varepsilon = \frac{1}{\rho}\sum_{i=1}^{n} \rho_i \varepsilon_i$, specific internal energy of mixture as a whole; η, coefficient of thermal relaxation of mixture; ρ, mixture density; ρ_i^0, true density of ith fraction; $\rho_i = \alpha_i \rho_i^0$, reduced density of ith component; ρ_{*i}, constant of equation of state; χ, thermal conductivity coefficient of mixture. Subscripts and superscripts: 0, in nonperturbed medium; (1) and (2), for mixture parameters "on the left" and "on the right" of contact discontinuity.

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